CHAPTER 8

NETWORK OPTIMIZATION
การวิเคราะห์ข่ายงาน

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NETWORK OPTIMIZATION

เป็นการวิเคราะห์โครงข่าย เพื่อหาความสัมพันธ์ หรือกำหนดทิศทางการไหลจาก node เริ่มต้น ไปยัง node สุดท้าย ในการวิเคราะห์ข่ายงาน บางครั้งเราอาจสนใจประเด็นดังนี้

1. **การวิเคราะห์หาทางเดินสั้นที่สุด (Shortest-path problem)** ระหว่าง node เริ่มต้น และ node สุดท้ายของข่ายงาน ให้มีระยะทางที่สั้นที่สุด เช่น การหาเส้นทางขนส่งที่ใกล้ที่สุด การเลือกสร้างทางรถไฟ เลือกสร้างถนน
2. **ให้มีจำนวนโครงข่ายน้อยที่สุด (Minimize spanning tree problem)** เช่นใช้ในการวางแผนวางโครงข่ายโทรศัพท์ให้ครอบคลุมพื้นที่ทั้งหมด
3. **การหาปริมาณการไหลสูงสุด (Maximum flow problem)** เช่น การหาจำนวนน้ำมันที่จะไหลได้มากที่สุด ในการขนส่งทางท่อ จำนวนรถที่วิ่งได้มากที่สุดบนโครงข่ายถนนที่ได้วางแผนไว้ จำนวนวัตถุดิบที่จะได้รับมากที่สุดในโครงข่ายของ supplier ที่มีอยู่

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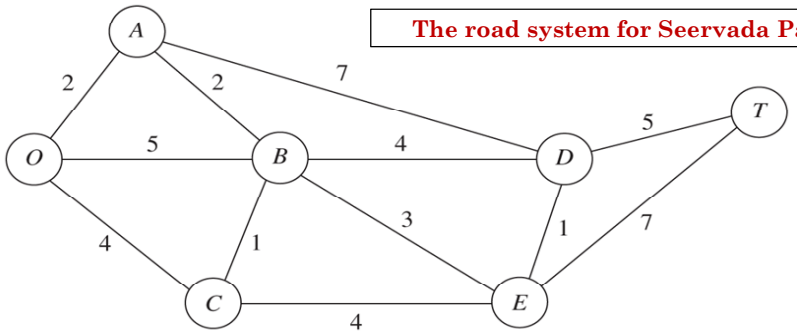
NETWORK OPTIMIZATION

- ***Nodes*** represented by ***circles***, present junction ***points*** connecting arcs.
- ***Arcs*** represented by ***lines***, connect nodes and present ***flow*** from one point to another.
- A decision needs to be made about the ***best way*** to flow or send something through a network.

COMPONENTS OF TYPICAL NETWORKS

Nodes	Arcs	Flow
Intersection	Roads	Vehicles
Airports	Air lanes	Aircraft
Switching points	Wires, channel	Messages
Pumping station	Pipes	Fluids
Work centers	Material-handling routes	Jobs



The road system for Seervada Park

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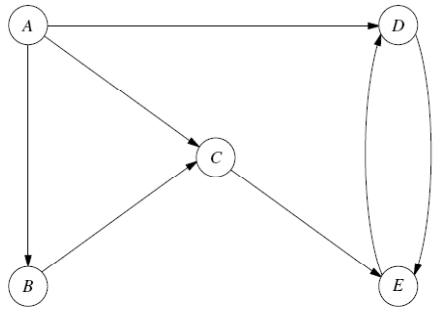
Network Optimization (4)

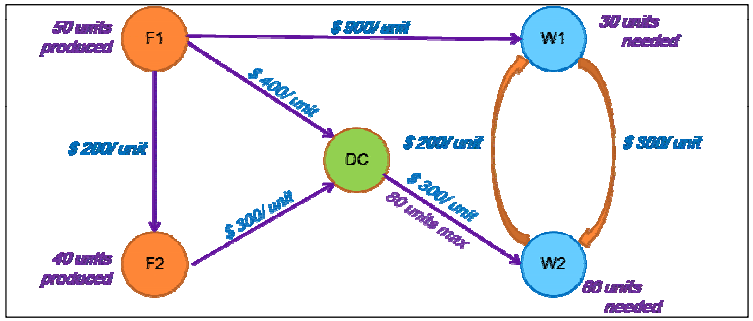
- Terminology of Networks (cont.)

FIGURE 9.2
The distribution network for Distribution Unlimited Co., first shown in Fig. 3.13, illustrates a directed network.



- Directed Path from A to E: A → B → C → E
- Undirected Path from A to E : B → C → A → D → E

A direct network



An example of direct network from chapter 3

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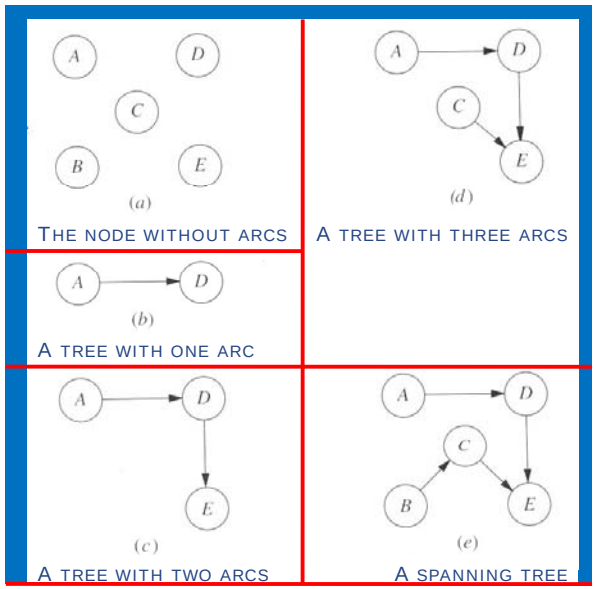
Example of network and tree



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(1) Shortest Path Problem

- **Shortest-Path Problem:**
  - Finding the shortest route from the starting node to each of the other nodes in the network.
  - "The shortest-path algorithm"

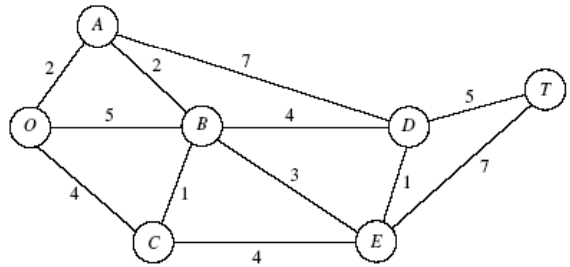
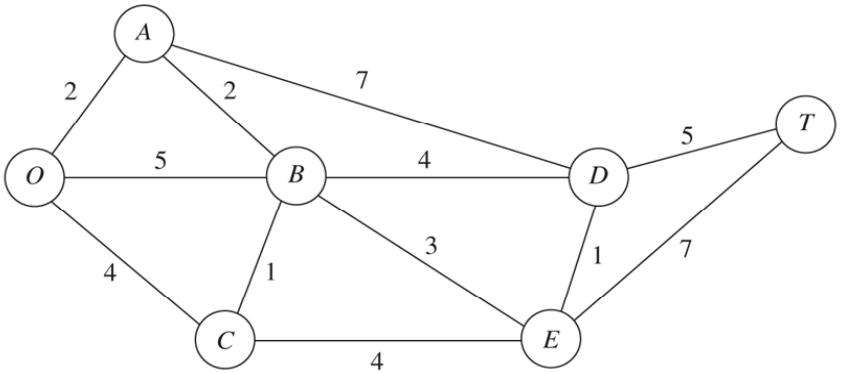


FIGURE 9.1
The road system for Seervada Park.

Shortest Path Problem



The road system for Seervada Park

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Network Optimization (10)

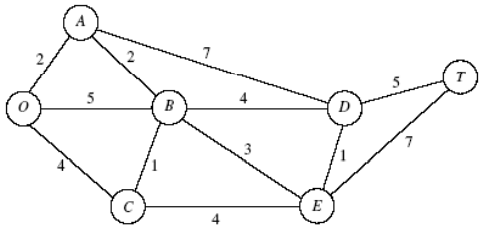
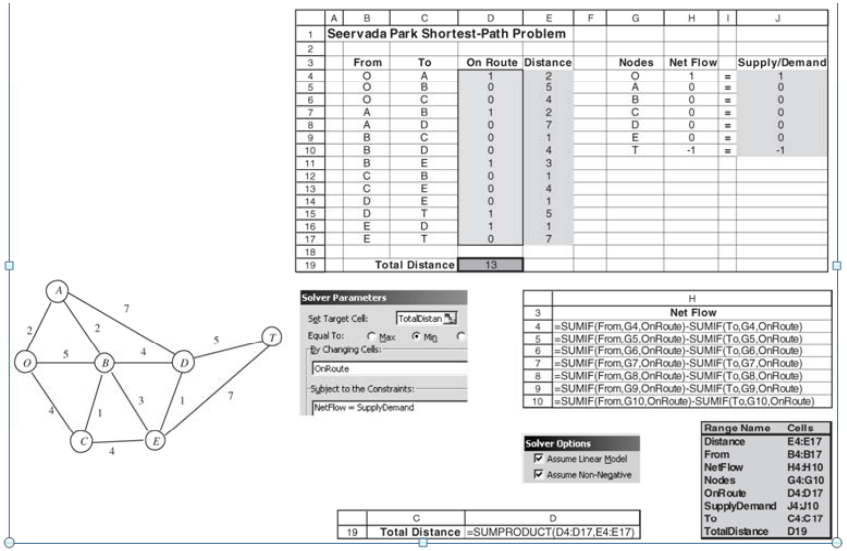


TABLE 9.2 Applying the shortest-path algorithm to the Seervada Park problem

n	Solved Nodes Directly Connected to Unsolved Nodes	Closest Connected Unsolved Node	Total Distance Involved	*n*th Nearest Node	Minimum Distance	Last Connection
1	*O*	*A*	2	*A*	2	*OA*
2, 3	*O* *A*	*C* *B*	4 2 + 2 = 4	*C* *B*	4 4	*OC* *AB*
4	*A* *B* *C*	*D* *E* *E*	2 + 7 = 9 4 + 3 = 7 4 + 4 = 8	*E*	7	*BE*
5	*A* *B* *E*	*D* *D* *D*	2 + 7 = 9 4 + 4 = 8 7 + 1 = 8	*D* *D*	8 8	*BD* *ED*
6	*D* *E*	*T* *T*	8 + 5 = 13 7 + 7 = 14	*T*	13	*DT*

Shortest Path Problem by EXCEL (11)



	A	B	C	D	E	F	G	H	I	J
1	Seervada Park Shortest-Path Problem									
2										
3		From	To	On Route	Distance		Nodes	Net Flow		Supply/Demand
4		O	A	1	2		O	1	=	1
5		O	B	0	5		A	0	=	0
6		O	C	0	4		B	0	=	0
7		A	B	1	2		C	0	=	0
8		A	D	0	7		D	0	=	0
9		B	C	0	1		E	0	=	0
10		B	D	0	4		T	-1	=	-1
11		B	E	1	3					
12		C	B	0	1					
13		C	E	0	4					
14		D	E	0	1					
15		D	T	1	5					
16		E	D	1	1					
17		E	T	0	7					
18										
19			Total Distance	13						

Solver Parameters
Set Target Cell: TotalDistan
Equal To: Max Min
By Changing Cells: OnRoute
Subject to the Constraints: NetFlow = SupplyDemand

	H
3	Net Flow
4	=SUMIF(From,G4,OnRoute)-SUMIF(To,G4,OnRoute)
5	=SUMIF(From,G5,OnRoute)-SUMIF(To,G5,OnRoute)
6	=SUMIF(From,G6,OnRoute)-SUMIF(To,G6,OnRoute)
7	=SUMIF(From,G7,OnRoute)-SUMIF(To,G7,OnRoute)
8	=SUMIF(From,G8,OnRoute)-SUMIF(To,G8,OnRoute)
9	=SUMIF(From,G9,OnRoute)-SUMIF(To,G9,OnRoute)
10	=SUMIF(From,G10,OnRoute)-SUMIF(To,G10,OnRoute)

Solver Options
Assume Linear Model
Assume Non-Negative

Range Name	Cells
Distance	E4:E17
From	B4:B17
NetFlow	H4:H10
Nodes	G4:G10
OnRoute	D4:D17
SupplyDemand	J4:J10
To	C4:C17
TotalDistance	D19

	C	D
19	Total Distance	=SUMPRODUCT(D4:D17,E4:E17)

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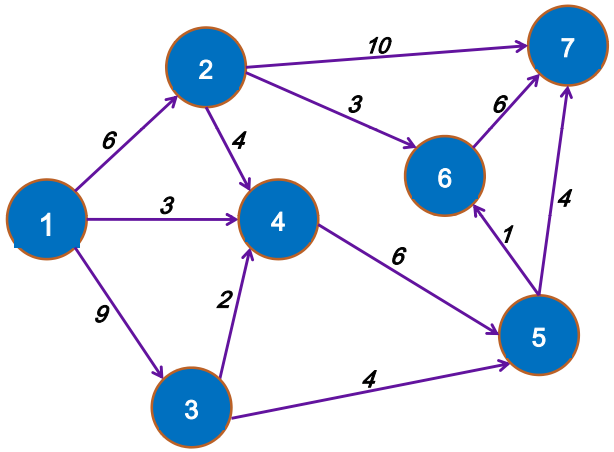
Shortest path problem (Exercise 1)



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SHORTEST PATH PROBLEM (APPLICATION: EX 2)

การทดแทนเครื่องมือ (Equipment Replacement)

บริษัทได้ทำการวางแผนการทดแทนเครื่องมือสำหรับ 5 ปีต่อไป โดยบริษัทสามารถคาดคะเนค่าใช้จ่ายในการซื้อเครื่องมือ ต้นปีที่ i และจะขายที่ต้นปีที่ j ได้โดยกำหนดเป็น C_{ij} จงหานโยบายที่ดีที่สุดในการทดแทนเครื่องจักร

$C_{12} = 700$	$C_{13} = 1400$	$C_{14} = 1700$
$C_{23} = 900$	$C_{24} = 1300$	$C_{25} = 1700$
$C_{34} = 1000$	$C_{35} = 1200$	$C_{36} = 2000$
$C_{45} = 800$	$C_{46} = 1200$	$C_{56} = 700$

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(2) THE MINIMUM SPANNING TREE PROBLEM

Some Applications

1. Design of telecommunication network
2. Design of a lightly used transportation network to minimize the total cost of providing links (rail, road)
3. Design of a network of high voltage electrical power
4. Design of a network of wiring on electrical equipment (digital computer)
5. Design of a network of pipelines to connect a number of locations

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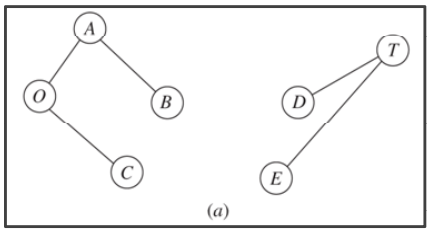
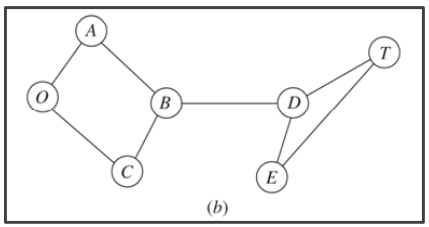
(2) THE MINIMUM SPANNING TREE PROBLEM

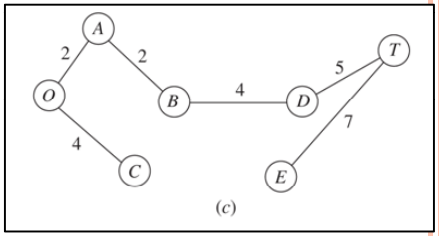
- Minimum Spanning Tree Problem
  - Once the arcs have been selected, they can provide a plan for setting up a system in which ***every node can communicate with every other nodes along some path*** connecting them.
  - This problem is to find the network with ***n*** node that requires only ***(n-1)*** links to provide a path between each pair of nodes with no undirected cycle.

THE MINIMUM SPANNING TREE PROBLEM



Not a Spanning Tree





Spanning Tree

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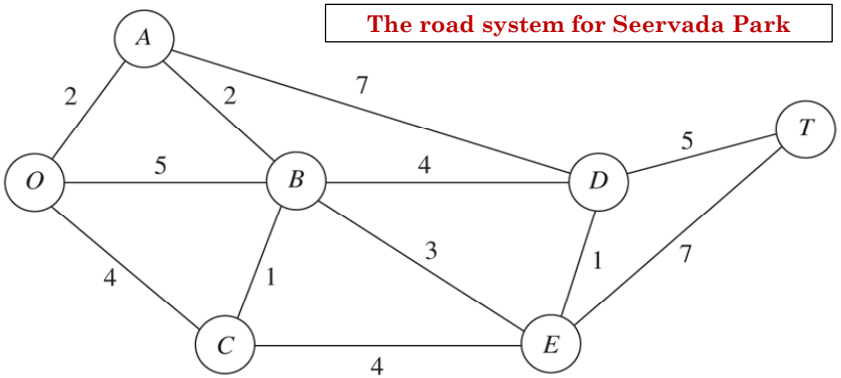
THE MINIMUM SPANNING TREE PROBLEM

The road system for Seervada Park



The network with *n* node require only *(n-1)* links to provide a path between each pair of nodes.

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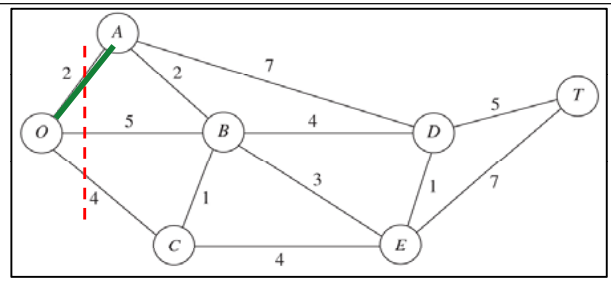
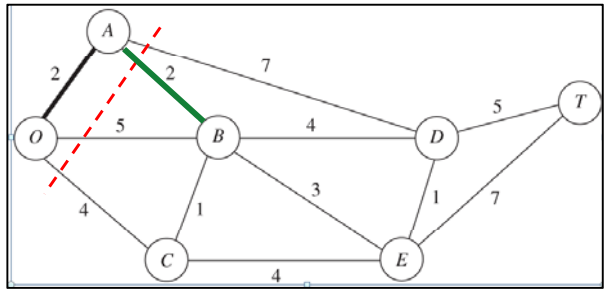
THE MINIMUM SPANNING TREE PROBLEM

Example: Install the phone line in Seervada Park

Start



Step-1



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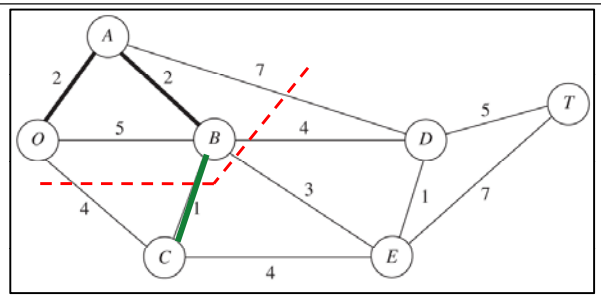
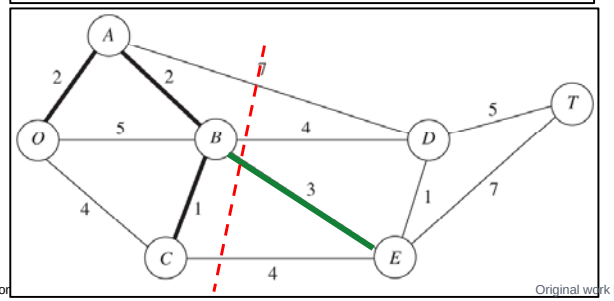
THE MINIMUM SPANNING TREE PROBLEM

Example: Install the phone line in Seervada Park

Step-2



Step-3



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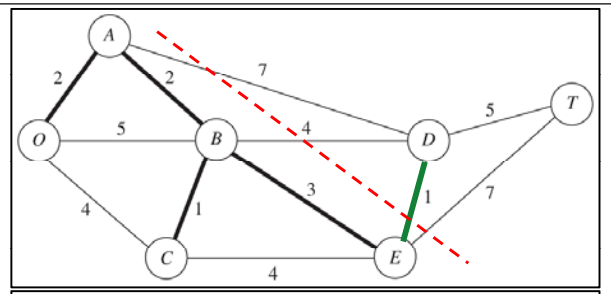
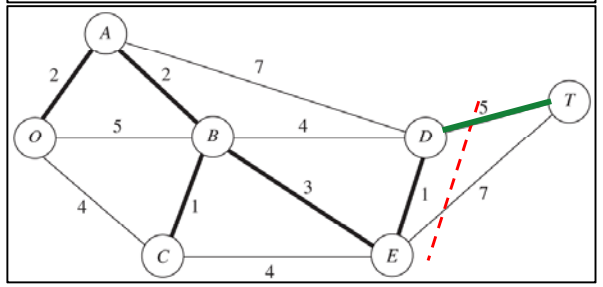
THE MINIMUM SPANNING TREE PROBLEM

Example: Install the phone line in Seervada Park

Step-4



Step-5



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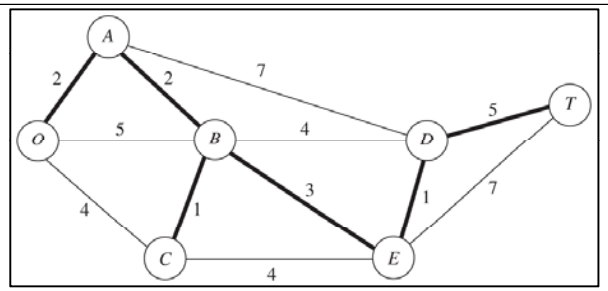
THE MINIMUM SPANNING TREE PROBLEM

Example: Install the phone line in Seervada Park



Step-6

Total length of the link is 14 miles = 2+2+1+3+1+5

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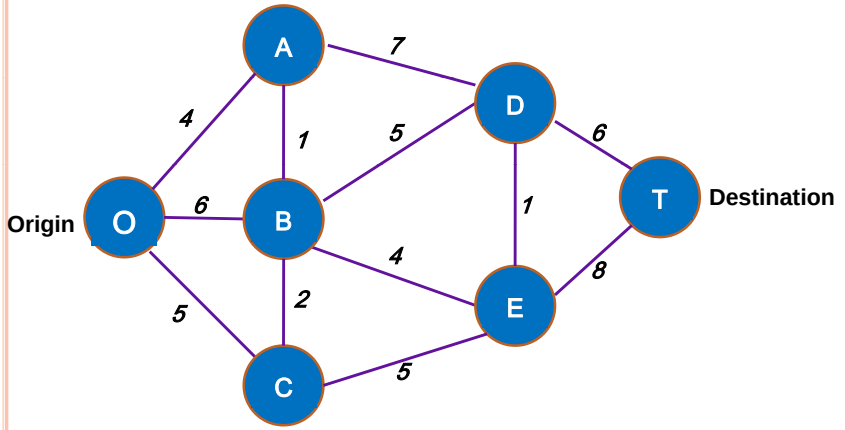
THE MINIMUM SPANNING TREE (EXERCISE 3)



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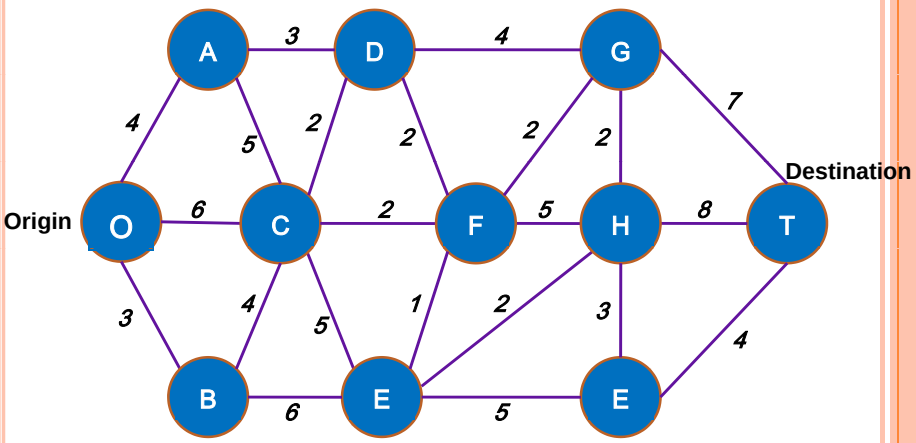
THE MINIMUM SPANNING TREE (EXERCISE 4)



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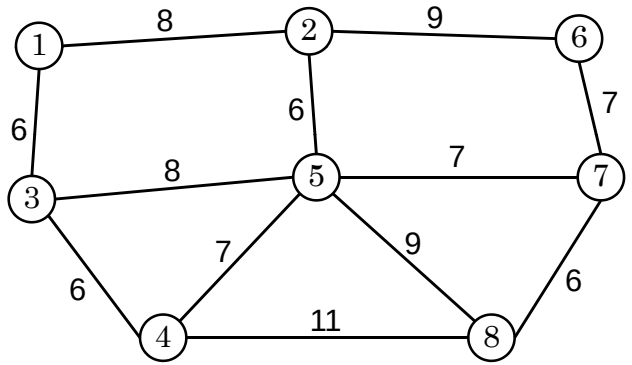
NETWORK OPTIMIZATION (18)

- Example: Solve this problem as Minimal Spanning Tree Problem



(3) THE MAXIMUM FLOW PROBLEM

เป็นปัญหาเกี่ยวกับการขนถ่ายสินค้า ข่าวสารข้อมูล หรือทรัพยากรใด ๆ จากโนดเริ่มต้น (Source Node) ไปยังโนดปลายทาง (Sink Node) โดยมีโนดระหว่างกลางเป็นโนดที่ขนถ่ายสินค้า

Applications:

1. Maximize flow through a company distribution network
2. Maximize flow through a company supply network
3. Maximize flow of the oil through a system of pipelines
4. Maximize flow of water through a system of aqueducts
5. Maximize the flow of vehicles through a transportation network

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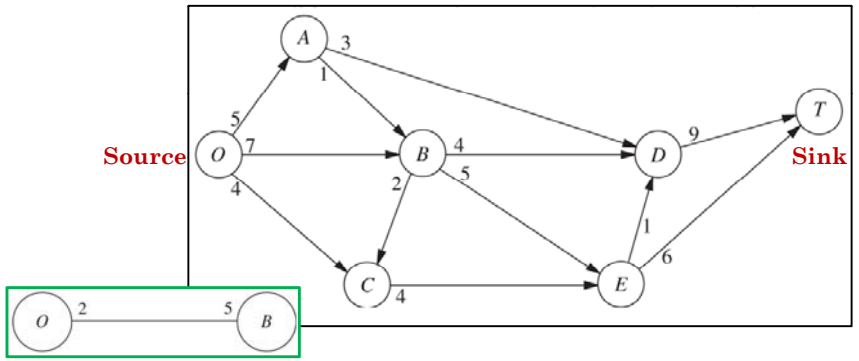
THE MAXIMUM FLOW PROBLEM

Example: Maximize tram trip from park entrance (Station 0) to the scenic wonder land (Station T)



แต่ละเส้นทางระหว่าง node จะมีการกำหนดจำนวนรอบสูงสุดที่จะสามารถวิ่งออกจาก park entrance ได้ต่อวัน ดังตัวเลขที่กำหนด

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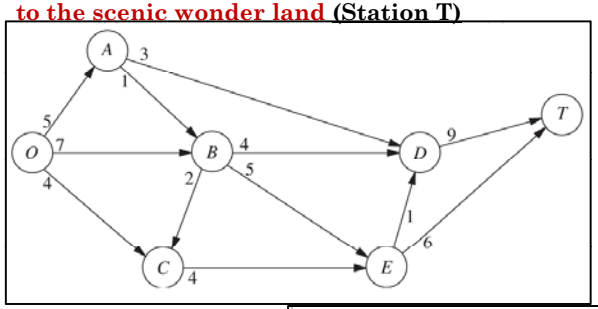
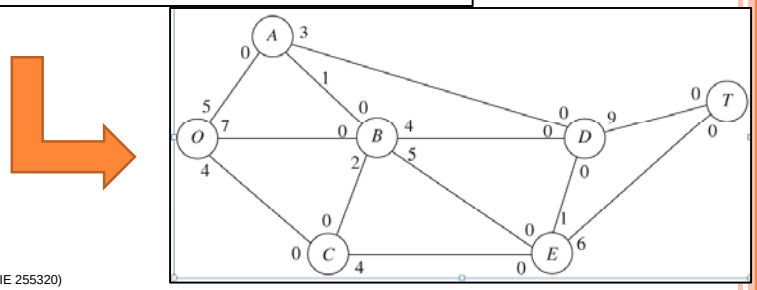
THE MAXIMUM FLOW PROBLEM (26)

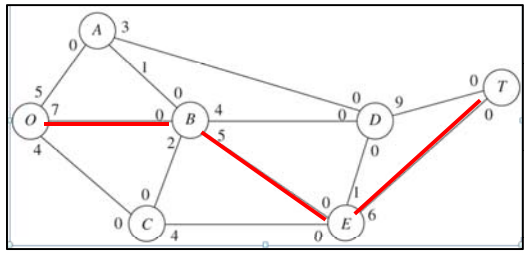
Example: Maximize tram trip from park entrance (Station 0) to the scenic wonder land (Station T)



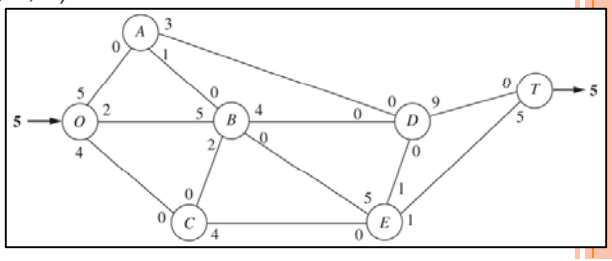


THE MAXIMUM FLOW PROBLEM (27)

- Iteration 0:

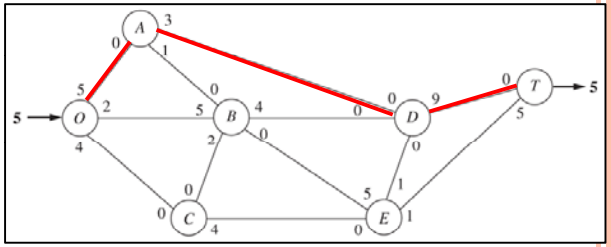


- Iteration 1: Pick O-B-E-T
  - Max Flow = Min (7, 5, 6) = 5

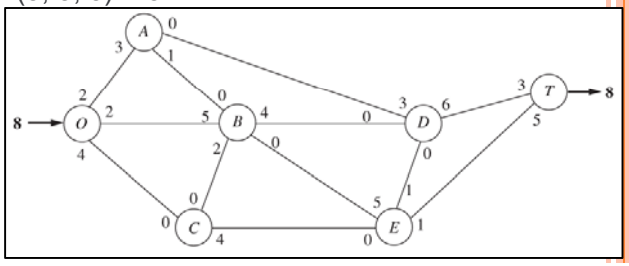


THE MAXIMUM FLOW PROBLEM (28)

- Iteration 1:

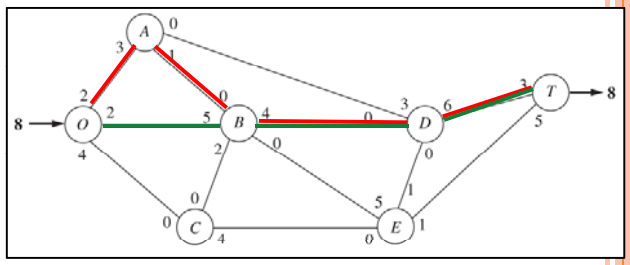


- Iteration 2: Pick O-A-D-T
  - Max Flow = Min (5, 3, 9) = 3

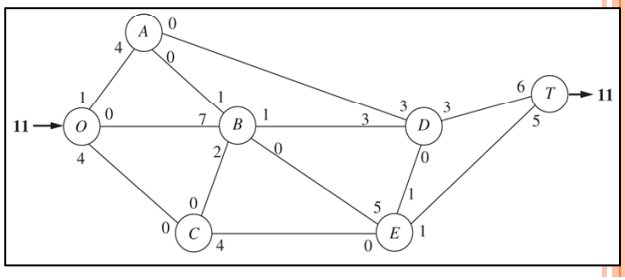


THE MAXIMUM FLOW PROBLEM (29)

- Iteration 2:

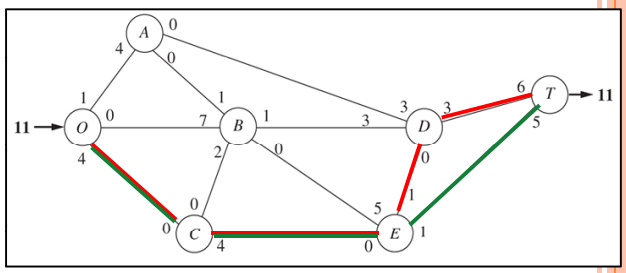


- Iteration 3+4: Pick O-A-B-D-T (Max Flow = 1) and O-B-D-T (Max Flow = 2)

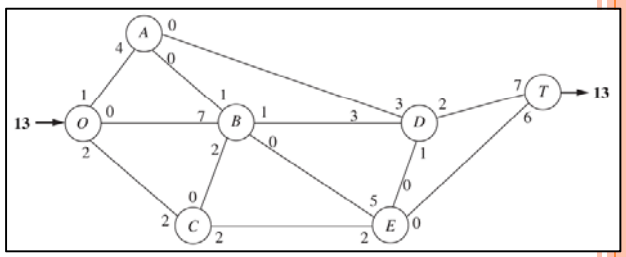


THE MAXIMUM FLOW PROBLEM (30)

- Iteration 3+4:

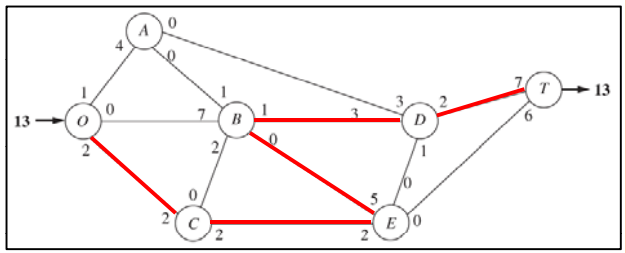


- Iteration 5+6: Pick O-C-E-D-T (Max Flow = 1) and O-C-E-T (Max Flow = 1)

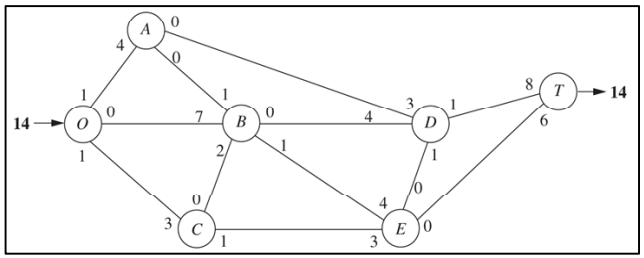


THE MAXIMUM FLOW PROBLEM

- Iteration 5+6:

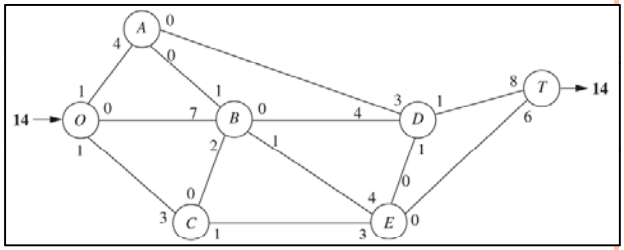


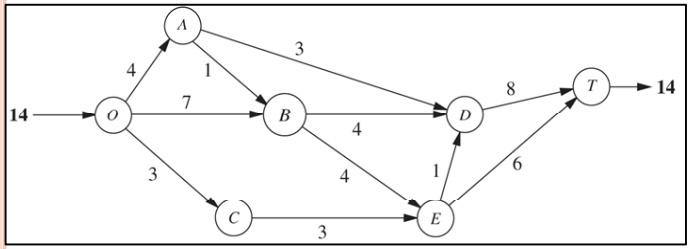
- Iteration 7: Pick O-C-E-B-D-T (Max Flow = 1)



THE MAXIMUM FLOW PROBLEM

- Optimal Solution:



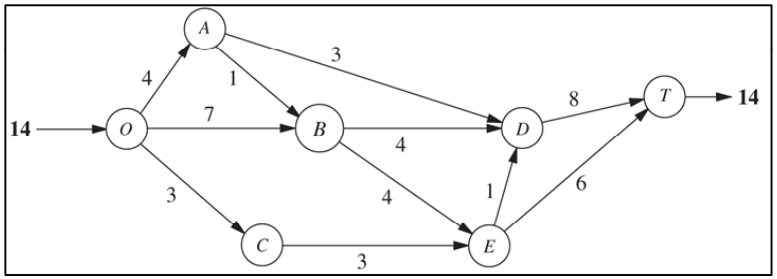


Maximum Flow = 14

THE MAXIMUM FLOW PROBLEM

Example: Maximize tram trip from park entrance (Station 0) to the scenic wonder land (Station T)

OPTIMAL SOLUTION



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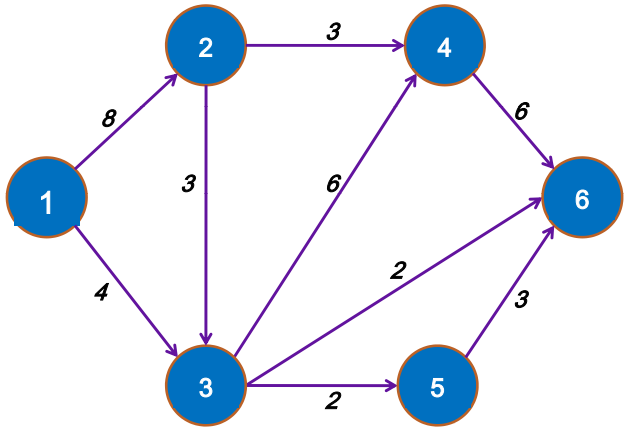
THE MAXIMUM FLOW PROBLEM (EXERCISE 5)



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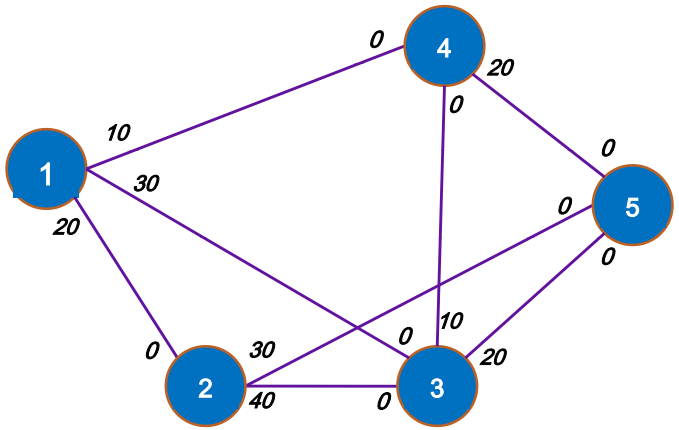
THE MAXIMUM FLOW PROBLEM (EXERCISE 6)



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